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Dynamical μ and MSSM

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Abstract: We present the idea that the vacuum can choose one pair of Higgs doublets by making the μ parameter a dynamical field called *massion*. The *massion* potential leading to the dynamical solution is suggested to arise from the small instanton interaction when the gauge couplings become strong near the cutoff scale M_s or M_P. One can construct supergravity models along this line. We also present an explicit example with a trinification model from superstring.

Keywords: Nonperturbative Effects, Superstring Vacua, Supersymmetric Standard Model, GUT.

Contents

1. Introduction

It is widely believed that the minimal supersymmetric standard model(MSSM) is the most probable immediate extension beyond the standard model(SM). It has three families of quarks and leptons, the SM gauge bosons, and their superpartners, and *one pair* of Higgs doublet superfields $\mathcal{H}_1$ and $\mathcal{H}_2$. The MSSM problem of obtaining just one pair is more constrained than the μ problem [1] or the doublet-triplet splitting problem [2]. For example, in the Z_3 orbifold compactification one can easily realize the doublet-triplet splitting [3], but the minimum number of doublets is three pairs. In this paper, we look for a possibility of dynamical solution of the MSSM problem, by promoting μ as a dynamical field.

The most well-known dynamical solution of a coupling constant is the axion solution of θ parameter [4]. In the θ vacuum, the euclidean space partition function determines the vacuum energy $E(\theta)$ as,

$$e^{-VE(\theta)} = \left| \int dA^a_\mu \det(\not{D} + M) \exp\left[-\frac{1}{4g^2} \int d^4x \, \mathrm{Tr}\, F_{\mu\nu} F_{\mu\nu} \right] \exp\left(\frac{i\theta}{16\pi^2} \int d^4x \, \mathrm{Tr}\, F\tilde{F} \right) \right| . \tag{1.1}$$

From eq. (1.1), one can show by using the Schwarz inequality that $E(0) \leq E(\theta)$. This is the basis of the Peccei-Quinn(PQ) mechanism making the vacuum choose $\theta \equiv a/F_a = 0$. For this mechanism to work, at tree level there is no potential of the axion field, i.e. it is a flat direction at tree level. The axion potential comes only from the one loop correction of the anomaly term.

Here, we ask a similar question on the μ term whether it can be understood dynamically. Actually, in supersymmetric models the determinental factor contains $(\not{D} + M_f)^2/(D^2 + M_b^2)$, revealing the information on the potential of the flat directional real scalar fields if it appears in the mass matrix. Thus, the mass matrix M in spontaneously broken supersymmetric models can be used for this purpose. We require that $\mu \equiv s$ does not have a potential at tree level so that μ can become a dynamical field *à la* the axion solution. Then, the effective potential with one-loop correction can be taken as

$$V = V_0 + \frac{1}{64\pi^2}\,\mathrm{Tr}(-1)^F M^4 \ln\frac{M^2}{\lambda^2} \tag{1.2}$$

where V_0 is the tree value and λ is the renormalization scale.[1] If supersymmetry is not broken, in the vacuum $V_0 = 0$ (in the global limit) and the one-loop correction vanishes. This has the needed property of the flat direction for s. This flat direction is *massion*, named for its role of determining the mass parameter of the Higgs doublets. This flat direction is lifted once supersymmetry is softly broken. Expressing the generic magnitude for the soft supersymmetry breaking as δ^2, the flat-direction lifting term is of order $\delta^2 M_P^2$.

However, the form (1.2) is not the one we expect toward a pair of light Higgs doublets since the minima generally do not choose a massless doublet. If the potential is of the form Det.M_f as suggested in [5], then a massless Higgsino doublet(s) and hence a massless pair(s) of Higgs bosons will follow. A possibility for this kind of determinental interaction is present if small scale instantons are important. Naively, one would expect that the small scale instantons would not affect the low energy physics significantly. But due to the possibility of packing a large number of instantons within a given volume if the instanton size is small (if the gauge coupling becomes strong at high energy), i.e. from the instanton size integration $\int d\rho/\rho^5$, the small scale instanton contribution to a small physical parameters can be significant. Indeed, the contribution of small scale QCD instantons was considered to the axion potential if QCD becomes strong at very high energy scale [6]. Of course, small scale instantons of other nonabelian groups can be important to the potential of almost flat directions. In this paper, we study such a possibility toward the potential of Higgs doublet fields.

We find that the useful small scale instantons toward the MSSM is the $q = 4$ instanton of the diagonal subgroup of $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ where $\mathrm{SU}(2)_L$ is the electroweak SU(2). This kind of embedding is possible in the trinification type and Pati-Salam type models.

In section 2, we show that the one loop potential (1.2) is of order $M_W^2 M_P^2$ and does not give a massless doublet, starting from many pairs of Higgs doublets. In section 3, we introduce a relatively strong force at a high energy scale. In section 4, we present the tangled instanton which does not emit ordinary quarks and leptons but emits Higgsinos. In section 5, we present this idea in a trinification model. Section 6 is a conclusion.

[1] We do not use the customary renormalization scale μ to avoid the confusion with the μ term.

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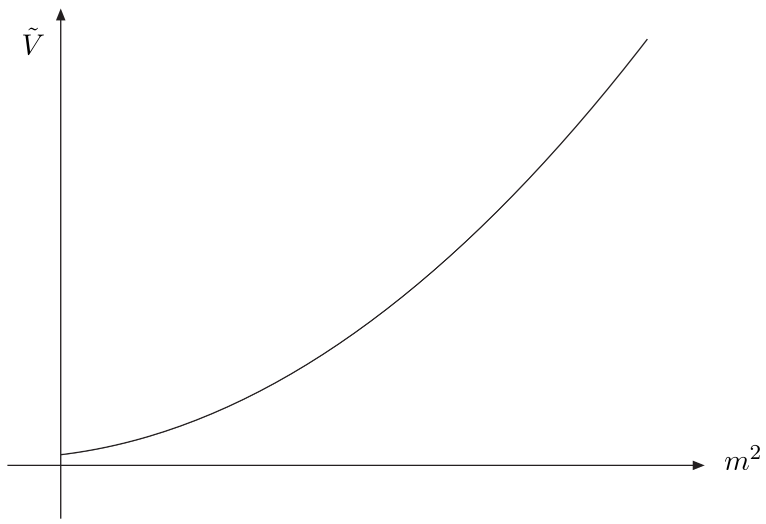

Figure 1: One pair of Higgs doublets.

2. One loop potential

Before introducing the massion s, let us consider the mass matrix M of (1.2). Our objective is to see if a pair(s) of Higgs doublets appear from a theory defined at a high energy scale. We do not assume the PQ symmetry at the electroweak scale. If we introduce the PQ symmetry, the MSSM spectrum does not allow the μ term. Also, we want to see if *only* one pair of Higgs doublets is allowed starting from huge Higgsino mass parameters at the high energy scale. Not assuming any specific symmetry here, the models discussed in ref. [7, 8] are not affected by our conclusion.

For one pair of chiral multiplets, S and $\bar{S}$, with a common mass splitting of δ^2, the effective potential is

$$V_1 = V_0 + \frac{2}{64\pi^2}\left[(m^2+\delta^2)^2 \ln\frac{(m^2+\delta^2)}{\lambda^2} - m^4 \ln\frac{m^2}{\lambda^2}\right] \equiv V_0 + \frac{2}{64\pi^2}\tilde{V}_1 \tag{2.1}$$

where

$$\tilde{V}_1 = \begin{cases} m^4 \ln\left(1+\dfrac{\delta^2}{m^2}\right) + (2m^2\delta^2+\delta^4)\ln\dfrac{m^2+\delta^2}{\lambda^2}\,, & \text{for } m^2 \neq 0\,, \qquad m^2 > \lambda^2 \\ \delta^4 \ln\dfrac{\delta^2}{\lambda^2}\,, \quad \text{for } m^2 = 0\,, \qquad \delta^2 > \lambda^2 & \end{cases} \tag{2.2}$$

Thus, the magnitude of V_1 is of order δ^2. If we make m^2 a dynamical variable, we can compare $\tilde{V}(m^2 = 0)$ with other values of $\tilde{V}$. We take the renormalization scale λ^2 less than $m^2 + \delta^2$ so that the bosonic contribution is positive. The shape of $\tilde{V}$ has the m^2 dependence as shown in figure 1. In this case, $m^2 = 0$ is the minimum of the one-loop potential. However, this property does not persist if there exist more than one pair of Higgs doublets.

We are interested in the following range of parameters,

$$\delta^2 \sim \text{TeV}^2\,, \qquad m^2 \sim M_P^2\,, \qquad m_{3/2} \sim \text{TeV} \tag{2.3}$$

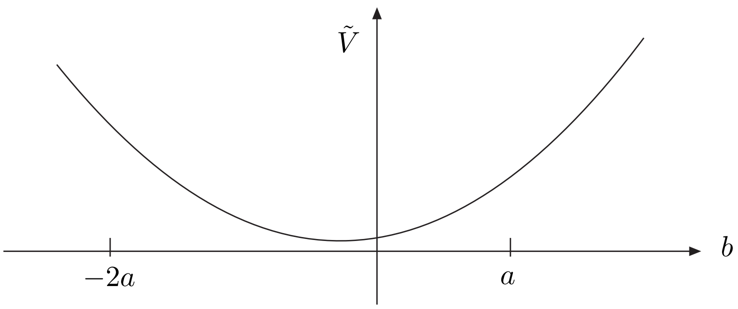

Figure 2: More than one pair of Higgs doublets.

where δ^2 is the mass splitting in spontaneously broken supergravity. Then, the A-term in V_0 has the contribution

$$AmH_1H_2 \to m_{3/2}v^2m$$

where v is the electroweak scale VEV. Thus, the A-term is negligible compared to δ^2m^2.

Now let us proceed to discuss a Higgsino mass matrix with an S_3 symmetry. One such matrix is

$$M_{\tilde{H}} = \begin{pmatrix} b & a & a \\ a & b & a \\ a & a & b \end{pmatrix} \tag{2.4}$$

which has eigenvalues of

$$m_{\tilde{H}} = b + 2a\,, \quad b - a\,, \quad b - a\,. \tag{2.5}$$

For this case, the shape of one-loop potential looks like figure 2. With the mass splitting parameter δ^2, the above potential is shifted up by $\mathrm{O}(\delta^2M^2)$ where M^2 is the mass parameter in the tree level potential, presumably of order the Planck scale. In supergravity, the vacuum value can be fine-tuned at $V = 0$ [10]. Let us take the sign of $a > 0$. In figure 2, a and δ are considered as fixed numbers and b is considered as a variable parameter, and we assumed $a^2 \gg \delta^2$ where $a^2 = O(M^2)$. Certainly, the minimum position in figure 2 is not near $b = a$ or $b = -2a$. Thus, even if massion is introduced, the one loop potential of eq. (2.1) is not guaranteeing a massless pair of Higgs doublets. Without a symmetry forbidding the Higgsino mass, one needs another interaction for massion to choose a massless pair of Higgs doublets.

3. The μ term as a field

3.1 The μ problem

The μ problem, "Why is μ so small compared to the GUT scale?", is a part of the MSSM problem for obtaining the MSSM spectrum. The μ term is the mass term for a vectorlike pair of fermions in the superpotential, i.e.

$$W_{\mu_X} = \mu_X\bar{X}X$$

where X and $\bar{X}$ are left-handed chiral superfields and each carries the charge conjugated gauge quantum numbers of the other. Thus, considering the gauge symmetry only, a non-vanishing μ_X is allowed and its magnitude is typically of order where the theory is written. In the MSSM spectrum at the electroweak scale, there exists a vectorlike pair of Higgsinos, $\tilde{H}_1$ and $\tilde{H}_2$. Thus, we expect a large μ_H, presumably at the GUT scale. But, there is a need to obtain one pair of light Higgsinos for the MSSM spectrum. This is the MSSM problem. If the MSSM results from a GUT, then a similar term μ_T would appear for a pair of vectorlike color triplet Higgsinos $\tilde{T}$ and $\tilde{\bar{T}}$. But the MSSM does not need the color triplet Higgsinos, which is called the doublet-triplet splitting problem. Thus, *the MSSM problem is the problem of obtaining a small μ_H only for one pair of Higgsinos but keeping the other μ_H's and all the μ_T's to remain large.*

Earlier suggestions for the solution of μ problem are firstly using some symmetries to forbid the μ_H term of $\mathcal{H}_1$ and $\mathcal{H}_2$ at the scale in consideration (but allow the μ_T term for color triplet Higgsinos at high energy scale) [7]. In particular, it should be forbidden in the superpotential. But, we need the μ_H term of order the electroweak scale to obtain a phenomenologically viable electroweak symmetry breaking. This is achieved by breaking the Peccei-Quinn(PQ) symmetry at the intermediate scale as suggested or at the scale leading to the gravitino mass [1]. Since the PQ symmetry breaking scale and the intermediate scale for supergravity are of the similar order, both of them give reasonable electroweak symmetry breaking. However, these suggestions do not give a rationale why only one pair of Higgs doublets remains light.

3.2 μ as a dynamical field

On the other hand, recently an ansatz was suggested so that that the MSSM problem can be understood dynamically [5]. In this spirit, we promote the *μ term as a dynamical field* [5]. The dynamical μ is called *massion.* For a vectorlike representation, the group singlet field(s) can contribute to the μ_X term, viz. $(\mu_X + s)\tilde{X}\tilde{\bar{X}}$, and hence we will use the mass parameter and the massion field s interchangeably.

Dynamical μ was considered before in the context of MSSM [8]. Our objective is different from theirs. We try to obtain *one massless pair* of Higgsinos while theirs is trying to see the electroweak effects if the PQ symmetry allows one light Higgsino pair.[2] If we introduce the PQ symmetry, it can be of a nonrenormalizable term [1] or a renormalizable term [8]. If the nonrenormalizable term is introduced, μ behaves as a parameter of the MSSM plus the phenomenology of the very light axion [9]. On the other hand, if it is introduced as the renormalizable term, then there appear a few corrections on the MSSM phenomenology as discussed in [8]. Even if one does not introduce the PQ symmetry, introducing just one pair of Higgs doublets at the supergravity level is allowable in the dynamical scenario.

It is important to see how the MSSM spectrum arises from a more fundamental theory such as from superstring. In this regard, we note that there has not appeared yet any

[2]In particular, the second paper in [8] uses the PQ symmetry heavily even if breaking of the PQ symmetry by renormalizable terms(S^2, S^3) is not absolutely forbidden.

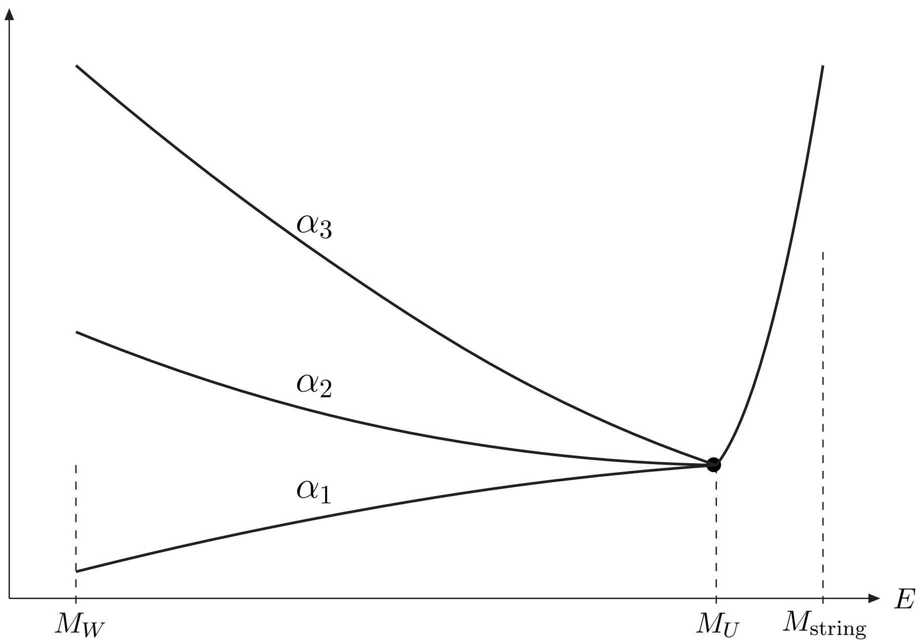

Figure 3: Gauge couplings above the GUT scale.

model allowing just one pair of Higgs doublets from superstring. So, it is important to see if there exists a mechanism choosing just one pair from many pairs. Here, it is not expected to use a global symmetry since a global symmetry is not the generic property of superstring. Without using a symmetry forbidding the Higgsino mass in the environment with many pairs of Higgs doublets, we can use the discussion of section 2. If there is no other contribution to the massion potential as we experience in the PQ mechanism with no potential of axion field, then it is not natural to obtain a massless pair of Higgs doublets from the extremum of the one-loop potential of the massion field. Therefore, we need a relatively strong force for this purpose. In fact, we observe that there exists such a possibility due to a large number of matter fields allowable above the GUT scale $\mathrm{M}_{\mathrm{GUT}}$. A large number of matter fields destroys the asymptotic freedom above $\mathrm{M}_{\mathrm{GUT}}$, and gauge couplings become stronger going above $\mathrm{M}_{\mathrm{GUT}}$. Neglecting Yukawa couplings, its behavior is shown in figure 3. We anticipate a situation that the nonabelian scale Λ where the interaction becomes strong is roughly the fundamental scale(or string scale) so that a perturbative discussion below the fundamental scale is possible.

3.3 Small-instanton generated potential

If the nonabelian scale Λ is around or above the string scale, the field theory calculation of the nonperturbative effect below the string scale is possible. In particular, the small-instanton solution, which is small in our TeV scale jargon but large at its nonabelian scale $\Lambda \geq M_s$, has the amplitude proportional to $e^{-8\pi^2/g^2(\lambda)}$ with $\lambda < M_s$. We will look for the situation where this nonperturbative effect is effective in determining a small parameter of the MSSM, i.e. the Higgsino mass parameter.

Before considering the asymptotically strong case, $N_f > 3N_c$, let us recapitulate the case $N_f < N_c$.

3.3.1 Case $N_f < N_c$

The nonabelian scale Λ is best understood in asymptotically free gauge models. So, for a moment consider asymptotically free nonabelian gauge theories, before we propose the coupling behavior of figure 3 above $\mathrm{M_{GUT}}$. Roughly, it is the scale where the coupling becomes strong.[3] Below the scale Λ, a QCD-like theory will show the confinement and chiral symmetry breaking. In *asymptotically free supersymmetric* QCD of SU(N_c), let us consider N_f pairs(or flavors) of left-handed superfields Q and $\bar{Q}$ with $N_f < N_c$. The classical lagrangian has the following global symmetries

$$SU(N_f)_L \times SU(N_f)_R \times \mathrm{U}(1)_A \times \mathrm{U}(1)_B \times \mathrm{U}(1)_R \tag{3.1}$$

with the following global quantum numbers of squarks and gauginos [11],

$$\begin{aligned} Q &: \left(N_f,\ \ 1,\ \ 1,\ \ 1,\ \ \frac{(N_f-N_c)}{N_f}\right) \\ \bar{Q} &: \left(1,\ \ N_f,\ 1,\ -1,\ \frac{(N_f-N_c)}{N_f}\right) \\ \text{gauginos} &: \ (0,\ \ \ 0,\ \ 0,\ \ 0,\ \ \ \ \ \ +1)\,. \end{aligned} \tag{3.2}$$

The $\mathrm{U}(1)_R$ quantum numbers are chosen so that it is anomaly free. The anomalous U(1) is just $\mathrm{U}(1)_A$. At a long distance limit, much larger than Λ^{-1}, the effective supersymmetric theory is parametrized by the squark VEVs, but it must respect the symmetries of (3.1). The symmetries of the effective interaction is given by the nonperturbative instanton effects whose symmetry is coming basically from 't Hooft determinental interaction [13]. The instanton amplitude is proportional to $e^{-8\pi^2/g^2(\lambda)} \simeq (\Lambda/\lambda)^{(3N_c-N_f)}$. Here, Λ is interpreted as obtained by integrating the N_f pairs of fermion zero modes. Therefore, the $\mathrm{U}(1)_A$ charge of $\Lambda^{(3N_c-N_f)}$ is interpreted as that of N_f pairs of squarks. One must have an appropriate power so that the $\mathrm{U}(1)_R$ symmetry is preserved. Thus, from the consideration of supersymmetry and global symmetries one obtains the effective superpotential as [12],

$$W_{\mathrm{eff}} = C_{N_c,N_f}\left(\frac{\Lambda^{3N_c-N_f}}{\mathrm{det}.Q\bar{Q}}\right)^{1/(N_c-N_f)} \tag{3.3}$$

where C_{N_c,N_f} is a constant. eq. (3.3) shows the runaway behavior of the squark fields at low energy for $N_f < N_c$.

Discussions on $N_f \leq 3N_c$ are summarized in [11].

3.3.2 Case $N_f > 3N_c$

On the other hand, if $N_f > 3N_c$ then it is asymptotically strong and the superpotential given in (3.3) does not make sense as an effective theory. At a larger separation, squarks behave more freely and the condensation of squarks is not anticipated. They behave more

[3] For QCD, Λ_{QCD} is a few hundred MeV.

like free squarks. It is known that the superpotential given above does not make sense. But the superpotential we wrote respects all the global symmetries. Suppose however that we interpret it as the effective superpotential of free squarks, generated by small-instantons. Then there results an inconsistency as shown below.

The scale of the small instanton is determined by the coupling strength at that small-instanton size. For supersymmetric nonabelian gauge theories, the one loop corrected coupling evolves as

$$\alpha(\Lambda) = \frac{\alpha}{1 + \frac{\alpha}{4\pi}(3N_c - N_f)\ln(\Lambda^2/\lambda^2)} \tag{3.4}$$

where $\alpha = \alpha(\lambda^2)$. Thus, the instanton amplitude at the scale Λ^2 is estimated as

$$e^{-8\pi^2/g^2(\Lambda^2)} = e^{-8\pi^2/g^2(\lambda^2)} e^{-\frac{1}{2}(3N_c - N_f)\ln(\Lambda^2/\lambda^2)} = e^{-2\pi/\alpha}\left(\frac{\Lambda}{\lambda}\right)^{N_f - 3N_c}. \tag{3.5}$$

If we assign the $\mathrm{U}(1)_A$ quantum number of $2N_f$ to $(\Lambda)^{N_f-3N_c}$ as before, we anticipate a superpotential, respecting the $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_A \times \mathrm{U}(1)_B$ symmetry as

$$W \to e^{-2\pi/\alpha} C_{N_f,N_c}\left(\frac{\Lambda}{\lambda}\right)^{N_f-3N_c}\left(\frac{1}{\mathrm{det}.Q\bar{Q}}\right) \to e^{-2\pi/\alpha}\left(\frac{\Lambda^{N_f-3N_c}}{\mathrm{det}.Q\bar{Q}}\right)^{\text{a power}}.$$

Considering the $\mathrm{U}(1)_R$ symmetry also, we expect

$$W = e^{-2\pi/\alpha}\lambda^{\frac{4N_f}{N_f-N_c}}\left(\frac{\Lambda^{N_f-3N_c}}{\mathrm{det}.Q\bar{Q}}\right)^{1/(N_f-N_c)}. \tag{3.6}$$

However, the vacuum does not exist with $\mathrm{SU}(N_f) \times \mathrm{U}(1)_A \times \mathrm{U}(1)_B \times \mathrm{U}(1)_R$ because det. $Q\bar{Q} = 0$. This arose by imposing supersymmetry and matching the global charges of the original and the effective theories. So the supersymmetry with the meson condensation is inconsistent with the asymptotically strong in the ultraviolet or asymptotically free in the infrared region. In fact, it was too far fetched to introduce a nonperturbatively generated VEVs for squark condensates where the theory is infrared-free.

Now, we may include the 't Hooft vertex directly, and introduces soft supersymmetry breaking with mass splitting of order δ^2. The 't Hooft vertex is shown in figure 4. The effect of the short distance instanton interaction is obtained by closing the quark loops and gluino loops with the insertion of their current masses, which is shown in figure 5. It gives a power of masses of the form

$$\propto m_{\tilde{G}}^{N_c} \cdot \mathrm{det}.M_Q\,.$$

where $m_{\tilde{G}}$ is the gluino mass scale. However, we must pick up a term of order δ^2, taking into account of the soft supersymmetry breaking. Diagonalizing the quark mass matrix, we have the contribution to the vacuum energy as[4]

$$V(\rho) \simeq \lambda^{4-N_f-N_c} m_{\tilde{G}}^{N_c} |m_1 m_2 \cdots m_{N_f}| \left(1 - \cos(\theta_{\tilde{G}} + \theta_1 + \theta_2 + \cdots + \theta_{N_f})\right) \tag{3.7}$$

[4]The quark masses are considered to be small compared to the inverse size, ρ^{-1}, of the instanton.

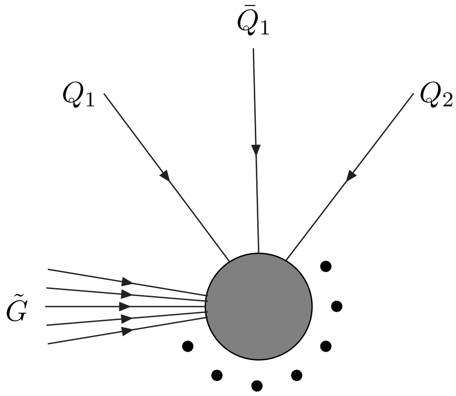

Figure 4: Instanton interaction. There are $2N_f$ quark lines and $2N_c$ gluino lines.

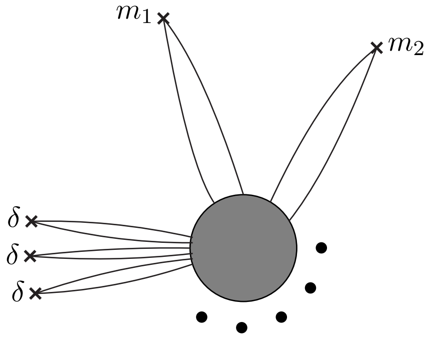

Figure 5: Closing the fermion loops by current masses, leading to the power $m^{N_f} m_{\tilde{G}}^{N_c}$.

where we fine-tuned the vacuum energy so that the SU(N_c) vacuum angle $\sum_i \theta_i = 0$ corresponds to the minimum. Thus, we can take the effective interaction by integrating out with the instanton size from the string scale M_s^{-1} to m_q^{-1},

$$V_{\text{eff}} \simeq \int_{1/M_s}^{1/m_q} \frac{d\rho}{\rho^5} D(\rho) \lambda^{4-N_f-N_c} m_{\tilde{G}}^{N_c} |m_1 m_2 \cdots m_{N_f}| \left(1 - \cos(\theta_{\tilde{G}} + \theta_1 + \theta_2 + \cdots + \theta_{N_f})\right) \tag{3.8}$$

where $D(\rho)$ is the density factor of the small-scale instantons. Taking $D(\rho)$ as [14]

$$D(\rho) = C_N \left(\frac{8\pi^2}{g^2(\rho^{-1})} \right)^{2N_c}, \qquad [C] = (\text{mass})^{-4} \tag{3.9}$$

for SU(N),

$$V_{\text{dilute}} \simeq \int_{1/M_s}^{1/m_q} \frac{d\rho}{\rho^5} D(\rho) e^{-2\pi/\alpha(\lambda)} \lambda^{4-N_f-N_c} m_{\tilde{G}}^{N_c} |m_1 m_2 \cdots m_{N_f}| \, (1 - \cos\theta) \tag{3.10}$$

$$\to e^{-2\pi/\alpha(\lambda)}\frac{C_N}{4}\left(\frac{2\pi}{\alpha(1/\bar\rho)}\right)^{2N_c}(M_s^4-m_q^4)\lambda^{4-N_f-N_c}m_{\tilde G}^{N_c}|m_1m_2\cdots m_{N_f}|(\cdots) \quad (3.11)$$

where $\theta=\theta_{\tilde G}+\theta_1+\theta_2+\cdots+\theta_{N_f}$, $\bar\rho$ is an appropriate average scale and in the last line we neglected the ρ dependence of λ. Note that $V_{\rm dilute}$ is negligible for $SU(2)_W$ and $SU(3)_c$ gauge groups since there exist light leptons and light quarks. For this mechanism to be useful at all, there should be an *additional* nonabelian group at high energy scale with its spectrum vectorlike. For this to be applicable to Higgsino pairs, Higgsinos must carry the vectorlike quantum numbers under this nonablian gauge group. This must be broken above the GUT scale $\mathrm{M_{GUT}}$. So, we may take $m_{\tilde G}\sim\mathrm{M_{GUT}}$. Hence the small instanton solution in the additional nonabelian group does not extends to infinity as in the case of unbroken nonabelian groups but extends only up to $M_{\rm GUT}^{-1}$. Nevertheless, the profile of the instanton solution of the additional nonabelian group for a scale $\ll M_{\rm GUT}^{-1}$ is very similar to that of the unbroken gauge group, which is understood below.

The determinental interaction $|m_1m_2\cdots m_{N_f}|$ in (3.11) takes a minimum when at least one quark mass vanishes.[5] Thus, we obtain degenerate vacua, corresponding to

$$\text{Case 1}: m_1=0,\ m_2=m_3=\cdots\neq 0 \quad (3.12)$$

$$\text{Case 2}: m_1=m_2=0,\ m_3=\cdots\neq 0\,, \quad (3.13)$$

etc.

The vacuum of Case 1 chooses one pair of Higgs doublets by cosmologically sliding down the massion field s, and the vacuum of Case 2 chooses two pairs of Higgs doublets, etc. Certainly, Case 1 belongs to the acceptable vacuum, leading to one pair of Higgs doublets.

The expression (3.11) is dominated by the smallest size instantons since the density of smaller size instantons is larger than the larger size instantons. If we included the ρ dependence of λ in the estimation of (3.11), the importance of the small size instantons in asymptotically strong theories is more conspicuous.

If we take $\alpha=1/25$ which is the value obtained at the scale $M_{\rm GUT}$ by extending the low energy couplings in SUSY GUTs, the GUT scale instantons would contribute as $10^{-50}M_{\rm GUT}^4$ which is utterly negligible compared to the supergravity parameter $M_P^2M_W^2$. But, for an illustration, consider the case of $e^{-2\pi/\alpha}\sim\frac{1}{10}$ at $\rho^{-1}=\frac12 M_s$, for which $\alpha=2.73$. Since loop corrections are expected to appear as powers of $\alpha/2\pi$, this value can be considered as the boundary value for a perturbative calculation. Then, setting every unknown mass parameter in eq. (3.11) as $\frac12 M_s$, eq. (3.11) gives the height of the potential as

$$\sim\frac{32\pi^4}{5}\delta^{N_c}M_s^{4-N_c}(4\pi)^{2(N_c-2)}\,.$$

If $N_c\geq 3$, the contribution of the small size instantons is negligible. On the other hand, if $N_c=2$, the instanton contribution can dominate, by a factor of 10^5-10^6, the one loop contribution of section 2 due to the $1/64\pi^2$ factor present in (2.1) and a large numerical factor in eq. (3.11). It is the dynamical realization of the doublet-triplet splitting, which

[5] Here, 'quark' corresponds to Higgsino.

was first put forward as an ansatz in ref. [5]. If Case (i) of (3.12) is chosen, then the MSSM results. Of course, this conclusion depends on the assumption of the strong gauge couplings at the string scale.

But the above type small-size instantons involving the SM quarks and leptons are completely negligible because the SM quark and lepton masses are less than TeV. One must employ another nonabelian gauge group which is broken at or above $\mathrm{M_{GUT}}$.

3.3.3 A supergravity toy model

Let us consider an $\mathrm{SU(2)}_R\times\mathrm{SU(2)}_L\times\mathrm{U(1)}_{Y_R}$ gauge theory with the following representations,

$$\mathcal{H} = (\mathbf{2},\mathbf{2}) \ :\ N_H \text{ flavors with } Y_R = 0 \tag{3.14}$$

$$R^c = (\mathbf{2},\mathbf{1}) \ :\ N_R \text{ flavors with } Y_R = +\frac{1}{2} \tag{3.15}$$

$$R = (\mathbf{2},\mathbf{1}) \ :\ N_R \text{ flavors with } Y_R = -\frac{1}{2} \tag{3.16}$$

$$l = (\mathbf{1},\mathbf{2}) \ :\ N_g \text{ flavors with } Y_R = -\frac{1}{2} \tag{3.17}$$

$$e^c = (\mathbf{1},\mathbf{1}) \ :\ N_g \text{ flavors with } Y_R = +1\,, \tag{3.18}$$

$$\text{and } N_g \text{ flavors of quarks} \tag{3.19}$$

where $N_g = 3$. The electromagnetic charge is

$$Q_{\rm em} = T_{R3} + T_{L3} + Y_R\,.$$

The $\mathrm{SU(2)}_R$ symmetry is broken at $\mathrm{M_{GUT}}$ with an $\langle R\rangle \sim\mathrm{M_{GUT}}$. $(R^c + R)$ is vectorlike and can be removed at $\mathrm{M_{GUT}}$. At low energy, we have the SM gauge group with the usual N_g lepton families. The small-scale $\mathrm{SU(2)}_L$ instanton interaction is negligible due to the lightness of quark and lepton masses. But the small-scale $\mathrm{SU(2)}_R$ instantons would emit $\mathcal{H}$, R^c and R. Naively, one would expect

$$\propto m^2_{\tilde G_R}\mathrm{det}.M_{\tilde{\mathcal{H}}} \tag{3.20}$$

where $m^2_{\tilde G_R}$ is of order $\mathrm{M_{GUT}}$ since $\mathrm{SU(2)}_R$ is broken at the GUT scale. For the $\mathrm{SU(2)}_R$ breaking, we need VEVs of R and R^c which are expected to be heavy at $\mathrm{M_{GUT}}$. This kind of mass insertions are understood in eq. (3.20). However, it should be further suppressed since supersymmetry restricts the SUSY mass splitting δ^2 appear in the potential. Thus, we will obtain instead

$$\propto \delta^2\mathrm{det}.M_{\tilde{\mathcal{H}}}\,. \tag{3.21}$$

Then following the previous argument, we obtain at least one light Higgs doublet. A nonrenormalizable superpotential of the following form

$$\frac{\langle R\rangle}{M_s}\mathcal{H}(le^c \text{ and } qd^c)\,, \qquad \frac{\langle R^c\rangle}{M_s}\mathcal{H}(qu^c)$$

can give mass to the SM fermions where $\mathcal{H}$ is the light Higgs doublet pair.

3.3.4 TeV scale Higgsino mass

Case (i) of (3.12) for $N_c = 2$ realizes one light Higgsino pair.[6] The color triplet Higgsinos are made superheavy not affected by the determinental interaction. But, for this scenario to be made successful, the one loop contribution will not spoil the condition of $m_1 \simeq 0$. If the potential of massion s from the determinental interaction is contaminated by other terms, one must ensure that the other terms do not spoil in choosing one massless pair of Higgs doublets. Suppose that they are composed of two terms, the determinental one from the small instanton contribution and the other from the one-loop contribution. Let us parametrize them by cosine potentials as

$$V = -A\left[\cos\left(\frac{s}{M}\right) + \epsilon\cos\left(\frac{s}{M} - \eta\right)\right]$$

where η is a mismatch phase between the two terms, and ϵ is expected to be of order 10^{-6}. With $\epsilon = 0$, one obtains $s = 0$ which corresponds to a zero Higgsino mass. For a small ϵ, the minimum of the potential occurs at $s/M \simeq \epsilon\eta$. For $\eta \leq 10^{-6}$, the Higgsino mass is less than $\sim 10^{-12}M \sim 100$ TeV. So an alignment of η close to 0 is needed to achieve a reasonable Higgsino mass. If only one massion couples to the massless Higgsino pair but not to the other pairs, then these two potentials are aligned, which is the case for the example discussed in section 5.

4. Instantons not emitting quarks and leptons

The instanton solution is a mapping from the group space S_3 to an euclidean space-time S_3. If we have two instantons one that of $\mathrm{SU}(2)_R$ and the other that of $\mathrm{SU}(2)_L$, both of them are good instanton solutions. These 'two' instantons carry both SU(2) group indices, but their centers can be different. Suppose that this composite instanton emits $\tilde{\mathcal{H}}$. If these instantons sit on top of each other, then the larger size instanton roughly sets the scale for emitting $\tilde{\mathcal{H}}$. If the distance of their separation is larger than the instanton sizes, then the separation distance roughly sets the scale for emitting $\tilde{\mathcal{H}}$, which is schematically shown in figure 6. These composite instantons have three sizes, two instanton scales $\rho_{R,L}$ and the distance a between them. The above composite instanton is helpful in introducing instantons with semi-simple groups.

Our interest is to find out some instantons by which $\tilde{\mathcal{H}}$ is emitted but quarks and leptons are not. Since the Higgsino in the trinification model and in the Pati-Salam model transforms as $(\mathbf{2},\mathbf{2})$ under $\mathrm{SU}(2)_R\times$ $\mathrm{SU}(2)_L$ while quarks and leptons do not transform in that way, we must utilize this $(\mathbf{2},\mathbf{2})$ property of $\tilde{\mathcal{H}}$.

The S_3 manifold in the group space is possible with SU(2). For an instanton solution embedding, let us call the relevant group $\mathrm{SU}(2)_{\rm inst}$. $\mathrm{SU}(2)_{\rm inst}$ can be embedded in $\mathrm{SU}(2)_R\times$ $\mathrm{SU}(2)_L$ either by identifying $\mathbf{2}_R \to \mathbf{2}_{\rm inst}$ and $\mathbf{2}_L \to \mathbf{2}_{\rm inst}$, or by identifying $\mathbf{2}_R \to \mathbf{2}_{\rm inst}$ and $\mathbf{2}_L^* \to \mathbf{2}_{\rm inst}$, i.e. by identifying $\mathrm{SU}(2)_R$ with $\mathrm{SU}(2)_L$ or with $\mathrm{SU}(2)_L^*$.[7] Let us call this

[6]Here, N_c corresponds to the nonabelian group $\mathrm{SU}(N_c)$.

[7]We encounter this kind of indentification in the spontaneosly broken $\mathrm{SU}(2)_R\times\mathrm{SU}(2)_L$ by the linkage Higgs fields $\langle(\mathbf{2},\mathbf{2}^*)\rangle$ or $\langle(\mathbf{2},\mathbf{2})\rangle$. For instanton solutions, however, we do not need these Higgs fields.

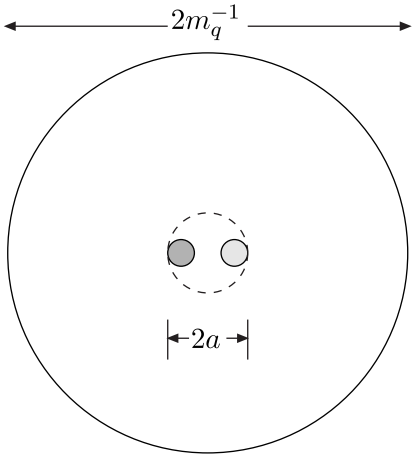

Figure 6: Two different instantons (darker grey and lighter grey ones) separated by $a \gg \rho_{R,L}$ within the space $\lambda^{-1} = m_q^{-1}$. A dashed boundary with a radius $\simeq a$ describes an effective size for the tangled instanton.

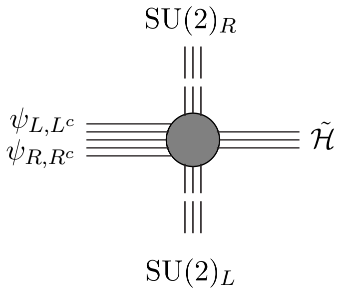

Figure 7: Tangled instanton with Pontryagin number 1. Solid lines correspond to quarks, and broken lines correspond to gluinos.

process of identification 'tangling' and the resulting instanton a 'tangled instanton'. A tangled instanton has three sizes, two instanton scales $\rho_{R,L}$ and the distance a between them. The largest among these is roughly the effective size of the tangled instanton, ρ_t. We can use ρ_t for the instanton amplitude we discussed in Subsection 3.3. For the gauge coupling, we adopt the most simple choice below: $g_R = g_L$ at the instanton scale. From now on, we do not use the concept of composite instantons. Just, the group property of $\mathrm{SU}(2)_{\mathrm{inst}}$ is important.

The Pontryagin number $q = 1$ of $\mathrm{SU}(2)_{\mathrm{inst}}$ corresponds to the original Pontryagin number 2 instanton since the gauge coupling of the diagonal subgroup is reduced by the factor $\frac{1}{\sqrt{2}}$ and hence the tunneling amplitude $\exp[-\frac{8\pi^2}{g^2}q]$ is the same in both interpretations.

Originally, N_R quarks ψ_{R,R^c} of $\mathrm{SU}(2)_R$, N_L quarks ψ_{L,L^c} of $\mathrm{SU}(2)_L$, and N_H Higgsino pairs $\tilde{\mathcal{H}}$ have the following flavor symmetry

$$SU(N_R) \times SU(N_R) \times SU(N_L) \times SU(N_L) \times SU(N_H) \tag{4.1}$$

where U(1)s are not written. The $\mathrm{SU}(2)_{\mathrm{inst}}$ instantons with Pontryagin number 1 must satisfy the global symmetry

$$SU(N_R+N_L+N_H)\times SU(N_R+N_L+N_H)\,.$$

Including gluinos, it is schematically shown in figure 7. Thus, we expect an instanton generated determinental interaction from $(\text{instanton})_R\times(\text{instanton})_L$ after integrating out the fermion lines as, identifying $\text{SU}(2)_R$ with $\text{SU}(2)_L$,[8]

$$\propto e^{i(\theta_R+\theta_L)}\text{det}.m_{\tilde G_R}\times \text{det}.m_{\tilde G_L}\times \text{det}.m_R\times\text{det}.m_L\times\text{det}.m_{\tilde{\mathcal H}}+\text{h.c.} \tag{4.2}$$

where h.c. is from $\overline{(\text{instanton})}_R\times\overline{(\text{instanton})}_L$. Masses are those of $\text{SU}(2)_R$ and $\text{SU}(2)_L$ gauginos, $\text{SU}(2)_R$ and $\text{SU}(2)_L$ quarks, and Higgsinos. From the tangled $(\text{instanton})_R\times\overline{(\text{instanton})}_L$ and its tangled anti-instanton, we would have

$$\propto e^{i(\theta_R-\theta_L)}\text{det}.m_{\tilde G_R}\times \text{det}.m^*_{\tilde G_L}\times \text{det}.m_R\times\text{det}.m^*_L+\text{h.c.} \tag{4.3}$$

which does not contain $\text{det}.m_{\tilde{\mathcal H}}$ because a chiral transformation of $\tilde{\mathcal H}$ rotates only a combination $\theta_R+\theta_L$. Certainly, these forms are consistent with the original global symmetry by assigning appropriate global transformation properties on the mass matrices. So far we considered the $\text{SU}(2)_{\text{inst}}$ instanton with Pontryagin number 1. The Pontryagin number 1 instantons emit doublets of $\text{SU}(2)_{\text{inst}}$.

How about the $\text{SU}(2)_{\text{inst}}$ instantons with Pontryagin number greater than 1? The Pontryagin number 2 $\text{SU}(2)_{\text{inst}}$ instatons are just two Pontryagin number 1 instantons which is a trivial extension of Pontryagin number 1 instantons.

The next simple representation of $\text{SU}(2)_R$(similarly for $\text{SU}(2)_L$) is the 3×3 representation which gives the Pontryagin number 4 instanton.[9] The Pontryagin index q is given by

$$q=\frac{1}{8\pi^2}\,\text{Tr}\int F_{\mu\nu}\tilde F_{\mu\nu}d^4x \tag{4.4}$$

where $\tilde F_{\mu\nu}=\frac12\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$. The well-known 2×2 representation of $\text{SU}(2)_{\text{inst}}$ gives $q=\pm1$ with

$$A_\mu=\frac{x^2}{x^2+\lambda^2}g^{-1}\partial_\mu g\,,\qquad g=\frac{x_4-ix_i\sigma_i}{r}=-\frac{i}{r}x_\mu\sigma_\mu\,,\qquad G^{-1}=\frac{i}{r}x_\mu\sigma_\mu \tag{4.5}$$

where σ^i are the ordinary Pauli matrices and the SU(2) generators are $T_i=\frac12\sigma_i$. For a general SU(2) representation T_i,

$$F_{\mu\nu}=\frac{4\lambda^2}{x^2+\lambda^2}T_{\mu\nu} \tag{4.6}$$

where

$$T_{ij}=i[T_i,T_j]\,,\qquad T_{i4}=-T_{4i}=-T_i \tag{4.7}$$

[8]It is straightforward to obtain determinental interactions for the case of identifying $\text{SU}(2)_R$ with $\text{SU}(2)^*_L$, which is not discussed here explicitly.

[9]The SU(2) embedding in SU(3) was considered in [15] where the Pontryagin number 4 instanton is constructed by four Pontryagin number 1 instantons. Here, we consider SU(2) and such a composition is not possible.

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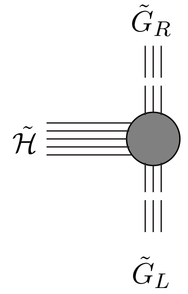

Figure 8: Tangled instanton with Pontryagin number 4. Solid lines correspond to quarks, and broken lines correspond to gluinos. The emitted quarks are only Higgsinos.

whence for a self-dual field $F_{\mu\nu} = \tilde{F}_{\mu\nu}$,

$$q = \frac{1}{6}\,\mathrm{Tr}\,T_{\mu\nu}T_{\mu\nu}\,. \tag{4.8}$$

For the doublet representation $T_i = \frac{1}{2}\sigma_i$, we obtain $q = 1$. For a triplet representation, $q = 4$.

In the tangling process of $\mathbf{3}_R$ and $\mathbf{3}_L$ instantons, i.e. by identifying $\mathrm{SU}(2)_R$ and $\mathrm{SU}(2)_L$, we have two Pontryagin number 4 intstantons in the original groups, i.e. the total Pontryagin number 8. However, in terms of $\mathrm{SU}(2)_{\mathrm{inst}}$ it is a Pontryagin number 4 instantion. As commented before, the tunneling amplitude is the same whichever interpretation we use since the $\mathrm{SU}(2)_{\mathrm{inst}}$ gauge coupling is smaller by a factor of $\frac{1}{\sqrt{2}}$. In this case, the triplets of $\mathrm{SU}(2)_R$ and $\mathrm{SU}(2)_L$ corresponding to the gluinos transform as a triplet of $\mathrm{SU}(2)_{\mathrm{inst}}$,

$$\mathbf{3}_R \text{ of } SU(2)_R \longrightarrow \mathbf{3}_{\mathrm{inst}} \text{ of } SU(2)_{\mathrm{inst}} \tag{4.9}$$

$$\mathbf{3}_L \text{ of } SU(2)_L \longrightarrow \mathbf{3}_{\mathrm{inst}} \text{ of } SU(2)_{\mathrm{inst}}\,. \tag{4.10}$$

The forms(instanton configuraion) of the triplet gauge field do not couple to a doublet (spinor) of SO(3) since spinors cannot be obtained by tensor products of the vector $\mathbf{3}$, i.e. we cannot write

$$(i\partial\!\!\!/ + \gamma^\mu A_\mu)\Psi = 0$$

for a 3×3 matrix $(A_\mu)^i_j$ $(i, j = 1, 2, 3)$ and Ψ_α $(\alpha = 1, 2)$. On the other hand, if Ψ is represented as a tensor product of the vector $\mathbf{3}$, for example Ψ_{ij}, then the above type of Dirac equation can be written. Thus, the Pontryagin number 4 instantons($\mathbf{3}_{\mathrm{inst}}$) do not emit $(\mathbf{2}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{2})$. In our case, the Higgsinos behave differently from the quark and leptons. They transform as a $\mathbf{3}$,

$$\mathbf{2}_R \times \mathbf{2}_L \longrightarrow \mathbf{3}_{\mathrm{inst}} + \mathbf{1} \text{ of } SU(2)_{\mathrm{inst}}\,. \tag{4.11}$$

Thus, in view of (4.9) and (4.10), $\mathrm{SU}(2)_R$ and $\mathrm{SU}(2)_L$ gluinos are emitted by $\mathrm{SU}(2)_{\mathrm{inst}}$ instantons, and in view of (4.11) $\tilde{\mathcal{H}}$ are emitted by $\mathrm{SU}(2)_{\mathrm{inst}}$ instantons. However, $\mathbf{2}_R = \psi_R, \psi_{R^c}$ and $\mathbf{2}_L = \psi_L, \psi_{L^c}$ are not emitted by the $\mathrm{SU}(2)_{\mathrm{inst}}$ instantons with Pontryagin number 4. Thus, the interaction we obtain from tangled instantons with Pontryagin number 4 with the identification of $\mathrm{SU}(2)_R$ with $\mathrm{SU}(2)_L$ is

$$\propto e^{i(\theta_R+\theta_L)}\mathrm{det.}m_{\tilde{G}_R} \times \mathrm{det.}m_{\tilde{G}_L} \times \mathrm{det.}m_{\tilde{H}} + \mathrm{h.c.} \tag{4.12}$$

which is schematically shown in figure 8. On the other hand, the interaction from tangled instantons with Pontryagin number 4 with the identification of $\mathrm{SU}(2)_R$ with $\mathrm{SU}(2)_L^*$ would be

$$\propto e^{i(\theta_R-\theta_L)}\mathrm{det.}m_{\tilde{G}_R} \times \mathrm{det.}m^*_{\tilde{G}_L} + \mathrm{h.c.}$$

where $\mathrm{det.}m_{\tilde{H}}$ is absent since a chiral rotation of $\tilde{\mathcal{H}}$ rotates only the combination of $\theta_R+\theta_L$.

Trinification models and Pati-Salam models give negligible contributions from figure 7 due to the highly-chiral nature of the SM quarks and leptons. Thus, among those involving $\tilde{\mathcal{H}}$, figure 8 is the dominant one. The small-scale instanton interaction we discussed in previous sections is attributed to the one coming from figure 8. Note that the eventual breaking of $\mathrm{SU}(2)_R$ at $\mathrm{M_{GUT}}$ does not change our argument since $\mathrm{SU}(2)_R\times\mathrm{SU}(2)_L$ is not broken at q where $M_{\rm GUT} < q < M_s$ and the size of small instantons we are considering is roughly $M_s^{-1} \ll M_{\rm GUT}^{-1}$.

In spontaneously broken supersymmetric models, the dominant instanton contribution will be at most of order $\delta^2\mathrm{det.}M$. Indeed, figure 8 gives this order, since the $\mathrm{SU}(2)_L$ gaugino mass contraction is of order δ^2 since $\mathrm{SU}(2)_L$ is broken at the electroweak scale and $\mathrm{SU}(2)_R$ gaugino mass contraction is of order $\mathrm{M_{GUT}}$ since $\mathrm{SU}(2)_R$ is broken at the GUT scale by $\langle(\mathbf{2},\mathbf{1})_{\pm1/2}\rangle$. However, $(\mathbf{2},\mathbf{1})_{\pm1/2}$ is not emitted by the $q=4$ instantons since it is a spinor of $\mathrm{SO}(3)_{\rm inst}$. In any case whether $(\mathbf{2},\mathbf{1})_{\pm1/2}$ is emitted(as in the example of Subsection 3.3) or not(as above), our idea for dynamical μ_H works only for this types of $\mathrm{SU}(2)_R\times\mathrm{SU}(2)_L$ theories where $\mathrm{SU}(2)_R$ is broken at a high energy scale. Trinification type models and Pati-Salam type models belong to this category. But the $\mathrm{SU}(5)_{\rm GG}$ and flipped SU(5) subgroups of E_6 cannot be made to work for the dynamical μ_H along the line we discussed here.

The tunneling amplitude for the Higgsino mass matrix is proportional to $e^{-32\pi^2/g^2_{\rm inst}}$ which has to be significant for the mechanism to work. If the massion has the flat direction except from this $q=4$ instanton contribution, then this will settle probably mass of one Higgsino pair near the electroweak scale.

5. Example with a trinification model

For an illustrative purpose, we adopt a discrete symmetry to tackle the problem with a reasonable simplicity. In section 2, an S_3 discrete symmetry has been used. String Z_3 orbifold compactification can have indeed this kind of the S_3 symmetry.

5.1 $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ as a subgroup of $SU(3)^3$

For an explicit discussion, from now on let us proceed with the trinification model of [16]. This model has enough independent directions so that the vacuum can choose a minimum of the potential. The gauge group is $\mathrm{SU}(3)_1\times\mathrm{SU}(3)_2\times\mathrm{SU}(3)_3$ and the quantum numbers of the spectrum is three times

$$\Psi_{\rm tri} = \mathbf{27}_{\rm tri} = (\mathbf{3}^*,\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}^*,\mathbf{3}) + (\mathbf{3},\mathbf{1},\mathbf{3}^*) \tag{5.1}$$

which is denoted as [17],

$$\begin{aligned}\Psi_l \to \Psi_{(\bar M,l,0)} &= \Psi_{(\bar 1,i,0)}(H_1)_{-1/2} + \Psi_{(\bar 2,i,0)}(H_2)_{+1/2} + \Psi_{(\bar 3,i,0)}(l)_{-1/2} + \\ &\quad + \Psi_{(\bar 1,3,0)}(N_5)_0 + \Psi_{(\bar 2,3,0)}(e^+)_{+1} + \Psi_{(\bar 3,3,0)}(N_{10})_0 \qquad (5.2)\\ \Psi_q \to \Psi_{(0,\bar l,\alpha)} &= \Psi_{(0,\bar i,\alpha)}(q)_{+1/6} + \Psi_{(0,\bar 3,\alpha)}(D)_{-1/3} \qquad (5.3)\\ \Psi_a \to \Psi_{(M,0,\bar\alpha)} &= \Psi_{(1,0,\bar\alpha)}(d^c)_{+1/3} + \Psi_{(2,0,\bar\alpha)}(u^c)_{-2/3} + \Psi_{(3,0,\bar\alpha)}(\bar D)_{+1/3}\,. \qquad (5.4)\end{aligned}$$

Here, we also show the standard model fields in brackets. The running indices are those of three SU(3)s, i.e. i for SU$(2)_W$ and α for SU$(3)_3$. SU$(2)_W$ is the subgroup of the second SU$(3)_2$ and QCD is the third SU$(3)_3$. The three types of representations are called humors: lepton-, quark-, and antiquark-humors with the obvious implication.

Let the trinification group be broken by $\langle N_{10}\rangle \sim M_s$ to,[10]

$$SU(3)_1 \times SU(3)_2 \times SU(3)_3 \to SU(2)_1 \times SU(2)_W \times SU(3)_c \times U(1)\,. \tag{5.5}$$

Here, SU$(2)_1$ is the SU$(2)_R$ and SU$(2)_W$ is the SU$(2)_L$ of section 4. So the term from figure 8 of tangled instantons emits pairs of $\tilde{\mathcal{H}}_1 = \Psi_{(\bar 1,i,0)}$ and $\tilde{\mathcal{H}}_2 = \Psi_{(\bar 2,i,0)}$. The interaction from figure 8 will involve det.$M_{\tilde{\mathcal{H}}}$.

For the trinification (5.1), the μ_H terms arise from the coupling of type Ψ_l^3, i.e.

$$N_{10}\mathcal{H}_1\mathcal{H}_2\,, \tag{5.6}$$

while the triplet μ_T term appears from the coupling of type $\Psi_l\Psi_q\Psi_a$, i.e.

$$N_{10}\bar D D\,. \tag{5.7}$$

Related to SU$(2)_{\rm inst}$, the notable difference of $\mathcal{H}_1$ and $\mathcal{H}_2$ from D and $\overline{D}$ is that the pair $\mathcal{H}_1$ and $\mathcal{H}_2$ transforms as $(\mathbf{2},\mathbf{2})$ under the SU$(2)_1\times$SU$(2)_W$ while D and $\overline{D}$ are SU$(2)_1$ and SU$(2)_W$ singlets. As commented before all Ds and $\overline{D}$s are removed at the GUT scale[11] since there is no tangled instanton interaction involving them only. So, SU$(2)_1$ and SU$(2)_W$ instantons emit $\tilde{\mathcal{H}}_1$ and $\tilde{\mathcal{H}}_2$ pairs and hence at least one pair of Higgs doublets out of three(or nine) pairs from three $\Psi_{\rm tri}$s(or $\mathbf{27}_{\rm tri} + \mathbf{27}_{\rm tri} + \overline{\mathbf{27}}_{\rm tri}$) [16] can remain light.

However, if the massion couplings to $\mathcal{H}_1\mathcal{H}_2$ and $D\overline{D}$ are of the same form, then the massion VEV determined by the small SU$(2)_1\times$SU$(2)_W$ instantons give the same eigenvalues to D and $\overline{D}$ pairs, and there will result a light D and $\overline{D}$. But, the origins of the Yukawa couplings of $\mathcal{H}_1$ and $\mathcal{H}_2$ and the Yukawa couplings of D and $\overline{D}$ are different, as pointed out in (5.6) and (5.7). So, if an S_3 symmetry is imposed, we expect the couplings of the form given in eq. (2.4), but with different sets of a and b of eq. (2.4), say a_H and b_H for $\mathcal{H}_1\mathcal{H}_2$ and a_D and b_D for $D\overline{D}$. So, a massless ratio of a_H and b_H for $\mathcal{H}_1\mathcal{H}_2$ does not necessarily lead to a massless ratio of a_D and b_D for $D\overline{D}$. Thus, when one Higgsino pair is made light by a tangled instanton with Pontryagin number 4, there does not necessarily result a light pair of D and $\overline{D}$. It is a dynamical solution of the MSSM problem.

[10] SU$(2)_1\times$SU$(2)_W\times$U(1) is broken down to the SM gauge group by $\overline{N_5}$ near $\mathrm{M_{GUT}}$ [16].

[11] In fact there is an additional alignment problem with $\mathcal{H}_1$ and $\mathcal{H}_2$, which will be commented below.

5.2 Example with three singlet fields

Consider just three copies of $\Psi_{\rm tri}$, neglecting $3(\mathbf{27}_{\rm tri}+\overline{\mathbf{27}}_{\rm tri})$, for the simplicity of discussion. The singlets generating the μ_H terms are N_{10}'s of (5.2),

$$\sum_{abc} f_{abc} N_{10}^{(a)} H_1^{(b)} H_2^{(c)} \tag{5.8}$$

where a, b, c are the flavor indices. Let us choose the Yukawa couplings $f_{abc} = f$ for $\mathcal{H}_1\mathcal{H}_2$ so that the calculation is simple. Namely, we choose $a_H = b_H$, and for this choice we expect in general $a_D \neq b_D$. The 3×3 Higgsino mass matrix becomes

$$M_{\tilde H} = \begin{pmatrix} f\langle N_{10}^{(1)}\rangle & f\langle N_{10}^{(3)}\rangle & f\langle N_{10}^{(2)}\rangle \\ f\langle N_{10}^{(3)}\rangle & f\langle N_{10}^{(2)}\rangle & f\langle N_{10}^{(1)}\rangle \\ f\langle N_{10}^{(2)}\rangle & f\langle N_{10}^{(1)}\rangle & f\langle N_{10}^{(3)}\rangle \end{pmatrix}. \tag{5.9}$$

Let $v_a \equiv \langle N_{10}^{(a)}\rangle$. Thus, there are three independent fields and they can settle at the minima near

$$\mathrm{Det}.M_{\tilde H} = -\frac{f^3}{2}(v_1+v_2+v_3)\left[(v_1-v_2)^2+(v_2-v_3)^2+(v_3-v_1)^2\right] = 0. \tag{5.10}$$

The eigenvalue x of $M_{\tilde H}$ satisfies

$$\begin{aligned} x^3 - f(v_1+v_2+v_3)x^2 + \frac{f^2}{2}[(v_1-v_2)^2+(v_2-v_3)^2+(v_3-v_1)^2]x - \\ -\frac{f^3}{2}(v_1+v_2+v_3)\left[(v_1-v_2)^2+(v_2-v_3)^2+(v_3-v_1)^2\right] = 0\,. \end{aligned} \tag{5.11}$$

The solutions of eq. (5.10) in the (v_1, v_2, v_3) space are

$$\text{(i) the plane } v_1+v_2+v_3 = 0\,, \quad \text{except the origin} \tag{5.12}$$

$$\text{(ii) the line } v_1 = v_2 = v_3\,, \quad \text{except the origin.} \tag{5.13}$$

Case (i) gives one pair of massless Higgsinos and Case (ii) gives two pairs of massless Higgsinos. Therefore, Case (i) allows the MSSM spectrum with one pair of Higgs doublets at low energy. Cosmologically, it is more probable for an arbitrary initial set of (v_1, v_2, v_3) to find the plane configuration before to find the line configuration. Thus, one light pair of Higgs doublets is expected to be chosen cosmologically.

Let us discuss Case (i) only, with $v_3 = -v_1 - v_2$. The mass eigenstates are

$$m_1 = 0: \ \psi^{(0)} = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{5.14}$$

$$m_\pm = \pm f\sqrt{\frac{3}{2}\left[v_1^2+v_2^2+(v_1+v_2)^2\right]} \tag{5.15}$$

$$\psi^{(+)} \propto \begin{pmatrix} v_1+2v_2 \\ -m_+ + v_1 - v_2 \\ m_+ - 2v_1 - v_2 \end{pmatrix} \tag{5.16}$$

$$\psi^{(-)} \propto \begin{pmatrix} v_1 + 2v_2 \\ m_+ + v_1 - v_2 \\ -m_+ - 2v_1 - v_2 \end{pmatrix}. \tag{5.17}$$

For Case (i), let the massions are chosen as

$$S_0 = \frac{1}{\sqrt{3}} \left(N_{10}^{(1)} + N_{10}^{(2)} + N_{10}^{(3)} \right) \tag{5.18}$$

$$S_{+,-} = \text{orthogonal to } S_0\,. \tag{5.19}$$

Thus, we may write the mass terms Higgsinos $\psi_i^{(0)} = \tilde{H}_i$ as

$$\propto S_0 \psi_1^{(0)} \psi_2^{(0)} + \cdots \tag{5.20}$$

where $\cdots$ does not contain S_0.

Since $S^{(0)}$ appears only with the massless Higgsino in the mass matrix, we can study its dependence on mass matrix easily even we include the one loop correction of (2.2). Adding eqs. (4.12) and (2.2), we can pick up the S_0 dependence from the $m^{(0)}$ eigenvalue which is zero at the $\mathrm{M_{GUT}}$ scale, which is parametrized as

$$V(S_0) \propto \delta^2 [m^{(0)}]^2 + \gamma \left\{ \delta^2 [m^{(0)}]^2 + 2\delta^2 [m^{(0)}]^2 \ln \frac{[m^{(0)}]^2 + \delta^2}{\lambda^2} \right\} \tag{5.21}$$

where we set δ^2 in eqs. (4.12) and (2.2) the same. The ratio of the overall interaction strengths is γ. Minimization of $V(S_0)$ leads to a nonzero $m^{(0)}$ at a value of order δ possibly corrected by a logarithmic factor. This is because $V(S_0)$ does not contain any large number except by a logarithmic factor of λ.

5.3 Massion coupling with S_3 symmetry

To show the form (5.20), it is convenient to use the tensor product table of S_3 [18].[12] The S_3 representations are **1** and **2**. Thus, three components of Higgsinos can split into either three **1**s or a **1** and a **2**. The latter case is of our immediate concern. Let us consider S_3 with elements $\psi^{(a)}$ with $a = 1, 2, 3$. The S_3 representations are

$$\begin{aligned} \mathbf{1} &= (1^0) = \psi^{(0)} = \frac{1}{\sqrt{3}} \left(\psi^{(1)} + \psi^{(2)} + \psi^{(3)} \right) \\ \mathbf{2} &= \begin{pmatrix} 2^+ \\ 2^- \end{pmatrix} = \begin{pmatrix} \psi^{(+)} \\ \psi^{(-)} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} \psi^{(1)} + \omega \psi^{(2)} + \omega^2 \psi^{(3)} \\ \psi^{(1)} + \omega^2 \psi^{(2)} + \omega \psi^{(3)} \end{pmatrix}. \end{aligned} \tag{5.22}$$

where ω is a cube root of unity, $e^{2\pi/3}$ (or $e^{-2\pi/3}$). The tensor product of **2** is

$$\mathbf{2} \times \mathbf{2} = \mathbf{2} + \mathbf{1} + \mathbf{1}'\,. \tag{5.23}$$

[12] The permutation symmetry has been extensively used before in particle physics. Some references are [19].

The doublet combination of (5.23) transforms under an S_3 operation as

$$\begin{pmatrix} \psi_1^{(+)}\psi_2^{(+)} \\ \psi_1^{(-)}\psi_2^{(-)} \end{pmatrix} \longrightarrow \begin{pmatrix} \omega^2\psi_1^{(-)}\psi_2^{(-)} \\ \omega\psi_1^{(+)}\psi_2^{(+)} \end{pmatrix} = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix} \begin{pmatrix} \psi_1^{(+)}\psi_2^{(+)} \\ \psi_1^{(-)}\psi_2^{(-)} \end{pmatrix} \tag{5.24}$$

where the 2×2 matrix in the last equation is a member of S_3 generators on doublets. On the other hand, the singlet couplings are

$$2^+\cdot 2^- + 2^-\cdot 2^+ = \mathbf{1}\,, \qquad 2^+\cdot 2^- - 2^-\cdot 2^+ = \mathbf{1}'\,. \tag{5.25}$$

For the doublet to obtain mass, it must couple to the doublet components among three $S^{(a)}$: $S^{(+)} = \frac{1}{\sqrt{3}}(S^{(1)}+\omega S^{(2)}+\omega^2 S^{(3)})$ or $S^{(-)} = \frac{1}{\sqrt{3}}(S^{(1)}+\omega^2 S^{(2)}+\omega S^{(3)})$. Thus, inverting this expression for $a=1,2,3$, we obtain $S^{(a)} = \frac{1}{\sqrt{3}}(S^{(0)}+\cdots)$, where $\cdots$ contain only $S^{(+)}$ and $S^{(-)}$. Suppose the doublet in the r.h.s. of (5.23) couples to a doublet of $S^{(a)}$ to give the combination $\mathbf{1}$ of (5.25). Then, we obtain

$$S^{(+)}(\psi_1^{(+)}\psi_2^{(+)}) + S^{(-)}(\psi_1^{(-)}\psi_2^{(-)})\,.$$

This is a proper form for the diagonalized Yukawa couplings. It should be such that $\langle S^{(\pm)}\rangle \propto m_\pm$. Namely, $S^{(0)}$ couples only to $\psi_1^{(0)}\psi_2^{(0)}$ as claimed in eq. (5.20) with $\langle S^{(0)}\rangle = 0$. The same conclusion is drawn from the combination $\mathbf{1}'$ of (5.25).

6. Conclusion

We considered the case where gauge couplings become asymptotically strong near the cutoff scale M_s or M_P. The asymptotically strong gauge coupling can arise due to the presence of a large number of matter fields below the compactification scale from the string compactification. In this high energy strong coupling regime, the small scale instanton contribution to the potential of the Higgs boson has been considered in this paper. We showed that the $q=4$ $\mathrm{SU}(2)_{\rm inst}$ instanton allows a determinental interaction of Higgsino mass matrix without the quark and lepton mass matrices as shown in figure 8,

$$\propto e^{i(\theta_R+\theta_L)}\mathrm{det.}m_{\tilde G_R} \times \mathrm{det.}m_{\tilde G_L} \times \mathrm{det.}m_{\tilde H} + \mathrm{h.c.}$$

Thus, the minimum of the potential is shown to make some of the Higgs boson pairs choose mass at zero which would be shifted to $\mathrm{O}(\delta^2)$ via the soft breaking of supersymmetry. This small size instantons can be made to work only for the gauge group $\mathrm{SU}(2)_R\times\mathrm{SU}(2)_L$ which can be a subgroup of the trinification group $\mathrm{SU}(3)^3$ or a subgroup of the Pati-Salam group $\mathrm{SU}(2)_R\times\mathrm{SU}(2)_L\times\mathrm{SU}(4)$. In this regard, we considered a trinification model where the *massion* is shown to couple only to the massless pair of the Higgs doublets. How many pairs of Higgs doublets are chosen to be light might be determined from the cosmological consideration. In the trinification example we considered, the vacuum with one massless pair is a plane while the vacuum with two massless pairs is a line, etc., in the (v_1, v_2, v_3) space, and hence cosmologically it is likely that the plane vacuum is more easily accessible, making just one massless pair for the Higgs doublets.

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